Love Affairs and Differential Equations

STEVEN H. STROGATZ
Harvard University
Cambridge, MA 02138

The purpose of this note is to suggest an unusual approach to the teaching of some standard material about systems of coupled ordinary differential equations. The approach relates the mathematics to a topic that is already on the minds of many college students: the time-evolution of a love affair between two people. Students seem to enjoy the material, taking an active role in the construction, solution, and interpretation of the equations.

The essence of the idea is contained in the following example.

Juliet is in love with Romeo, but in our version of this story, Romeo is a fickle lover. The more Juliet loves him, the more he begins to dislike her. But when she loses interest, his feelings for her warm up. She, on the other hand, tends to echo him: her love grows when he loves her, and turns to hate when he hates her.

A simple model for their ill-fated romance is

$$dr/dt = -aj, \qquad dj/dt = br,$$

where

$$r(t) = \text{Romeo's love/hate for Juliet at time } t$$
$$j(t) = \text{Juliet's love/hate for Romeo at time } t.$$

Positive values of r, j signify love, negative values signify hate. The parameters a, b are positive, to be consistent with the story.

The sad outcome of their affair is, of course, a neverending cycle of love and hate; their governing equations are those of a simple harmonic oscillator. At least they manage to achieve simultaneous love one-quarter of the time.

As one possible variation, the instructor may wish to discuss the more general second-order linear system

$$dr/dt = a_{11}r + a_{12}j$$
$$dj/dt = a_{21}r + a_{22}j,$$

where the parameters a_{ik} $(i, k = 1, 2)$ may be either positive or negative. A choice of sign specifies the romantic style. As named by one of my students, the choice a_{11}, $a_{12} > 0$ characterizes an "eager beaver"—someone both excited by his partner's love for him and further spurred on by his own affectionate feelings for her. It is entertaining to name the other three possible styles, and also to contemplate the romantic forecast for the various pairings. For instance, can a cautious lover ($a_{11} < 0$, $a_{12} > 0$) find true love with an eager-beaver?

Additional complications may be introduced in the name of realism or mathematical interest. Nonlinear terms could be included to prevent the possibilities of unbounded passion or disdain. Poets have long suggested that the equations should be nonautonomous ("In the spring, a young man's fancy lightly turns to thoughts of love"—Tennyson). Finally, the term "many-body problem" takes on new meaning in this context.